



## **Scaffolding Spatial Abilities in Integral Calculus**

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# Scaffolding Spatial Abilities in Integral Calculus

## Abstract

This NSF-IUSE exploration and design project began in fall 2018 and features cross-disciplinary collaboration between engineering, math, and psychology faculty to develop learning activities with hands-on models and manipulatives. We are exploring how best to design these activities to support learners' development of conceptual understanding and representational competence in integral calculus and engineering statics, two foundational courses for most engineering majors. A second goal is to leverage the model-based activities to scaffold spatial skills development in the context of traditional course content. As widely reported in the literature, well-developed spatial abilities correlate with student success and persistence in many STEM majors.

We provided calculus students in selected intervention sections taught by four instructors at three different community colleges with take-home model kits that they could reference for a series of asynchronous learning activities. Students in these sections completed the Purdue Spatial Visualization Test: Rotations (PSVT:R) in the first and last weeks of their course. We also administered the assessment in multiple control sections (no manipulatives) taught by the same faculty.

This paper analyzes results from fall 2020 through fall 2021 to see if there is any difference between control and intervention sections for the courses as a whole and for demographic subgroups including female-identifying students and historically-underserved students of color. All courses were asynchronous online modality in the context of the COVID-19 pandemic. We find that students in intervention sections of calculus made slightly larger gains on the PSVT:R, but this result is not statistically significant as a whole or for any of the demographic subgroups considered. We also analyzed final course grades for differences between control and intervention sections and found no differences.

We found no significant effect of the presence of the model-based activities leading to increased PSVT:R gains or improved course grades. We would not extend this conclusion to face-to-face implementation, however, due primarily to the compromises made to adapt the curriculum from in-person group learning to asynchronous individual work and inconsistent engagement of the online students with the modeling activities.

## Introduction

The EMARCS (Engaging with Modeling Activities for Representational Competence in STEM) project is in its fourth (no cost extension) year of a three-year NSF-IUSE exploration and design tier grant. This project is a collaboration between math and engineering faculty at Whatcom Community College working with psychology faculty at Western Washington University, all located in Bellingham, WA. We are working to integrate hands-on learning activities with 3D printed models and manipulatives in Integral Calculus and Engineering Statics instruction. The project emphasizes leveraging these activities to promote conceptual learning and embed spatial

skills training. Integral to the work is research to understand how students use the models as learning aids with a goal of using these observations to develop general activity design principles that may be applicable to a wider array of STEM courses. We presented the project rationale, goals and research questions along with the overall research design in 2020 [1].

One aspect of the project is to explore how students may be able to use their experiences with the manipulatives to improve their spatial visualization skills in the context of their Calculus or Statics course. The importance of spatial abilities for STEM majors in general is well-established [2]. Spatial abilities are malleable and can improve with targeted training [3], and there is evidence that such training can improve retention and graduation rates [4]. Colleges and universities widely use the Purdue Spatial Visualizations Test: Rotations (PSVT:R) [5] to measure students' spatial abilities upon entry and recommend supplemental spatial training to low-scoring students [6]. Female-identifying students generally enter college engineering programs with lower spatial abilities [7], so interventions targeting spatial skills may help improve their success rates and ultimately increase their percentage in the engineering workforce.

This paper focuses on the calculus intervention with emphasis on how presence of the models in course curriculum affects students' development of spatial abilities as they progress through the course. Integral calculus includes key prerequisite concepts and skills that are foundational to many engineering disciplines. Many of these concepts are spatial in nature and likely require well-developed visualization skills to understand [8]. One previous study showed that targeted spatial training in a separate concurrent course improved grades in a first-semester calculus course with content on differentiation and introductory integration [7]. A second, later study found no significant effect on course grades from a more limited intervention implemented within the course in either a second-quarter calculus course (integration techniques and applications) or a third-quarter course (series, vectors, multi-variable) [9].

## **Study Design**

We conducted this study in integral calculus courses taught by four different instructors at three community colleges in northwest Washington: Whatcom Community College (WCC), North Seattle College (NSC), and Tacoma Community College (TCC). This ten-week long quarter-system Calculus 2 course focuses on integration techniques and applications. All data collection occurred during fall 2020 – winter 2022 with asynchronous online learning as the only option at all three sites due to the respective colleges' operations plans amidst the COVID-19 pandemic. During the study, each instructor taught at least one control section and one intervention section, some ultimately teaching multiple sections with each condition over the study duration. For the intervention sections, we provided instructors with a scaled down take-home version of a series of models we initially developed as a basis for active learning in face-to-face instruction [10]. We reported last year on modifications we made to reduce cost and scale down parts so that we could continue this project in online courses by distributing a kit to each individual student. Student feedback on the hasty adaptation of the curriculum for online learning at home was not as positive as it had been for the in-class activities [11]. The model curriculum supports the

topics of volumes of revolution, volumes of similar cross-section, and centroids in 1D and 2D. Figure 1 shows a photo of the take-home kit that we distributed to students.



**Figure 1.** Take-home model kit distributed to students in the intervention sections.

One of the instructors in the study was the primary developer of the models and associated activity worksheets. We provided the other three participating faculty with a brief introduction to the curriculum in a series of videoconference meetings and ongoing support via one-to-one communication throughout the implementation period. The faculty participants were free to use the activity worksheets we provided, to modify them, or to develop their own approaches for incorporating the models into their respective courses. Two of the instructors in the study do not cover centroids as an integration application in their respective courses. This difference adds variability to the data set with respect to the scale of the intervention.

Note that the present intervention does not include specific targeted spatial skills training analogous to that used in the studies mentioned above. Rather, we hypothesize that access to the models might provide scaffolding that helps students engage with the more spatial-intensive topics (e.g. volumes of revolution) and, particularly for students entering the course with lower spatial ability, provide additional ways for them to think about the associated concepts as they work to build understanding. This hypothesis generates the following two research questions:

RQ1: Does work with the models lead to more improvements in students' spatial skills compared to traditional instruction?

RQ2: Does use of the models improve student success rates and outcomes in the course in which the intervention took place?

The present study analyzes PSVT:R results and final course grades to investigate RQ1 and RQ2 respectively.

### *Data Collection*

Instructors administered the PSVT:R as a timed online quiz in the first and last weeks of their respective courses using either the Canvas learning management system or WAMAP [12]. Students earned a nominal number of participation points as an incentive for completion. We also developed a series of multiple choice plus explain (MCE) concept check questions associated with the topics covered by the activities as part of this project [10]. Instructors assigned some of the MCE questions for practice opportunities of various forms before including others on course exams for all intervention sections and some of the control sections. We plan to publish results and analysis of the MCE data in a future paper. Instructors also reported final course grades to the research team. Students provided demographics information on an end-of-course survey that also provided opportunity for them to consent for their data to be included in this study.

We developed a web-based data management app (DMA) to collect the data for centralized analysis. The DMA filters for consent and then correlates the assessment data across multiple spreadsheet exports with students' demographic information collected through the survey. The DMA removes all identifying information before the course instructor shares their collated course data with the research team for centralized analysis. The data collection and consent procedures were all approved by the Whatcom Community College Institutional Review Board (IRB).

### *Participants*

Table 1 summarizes the numbers of students from the various instructors in the study who consented for their data to be included and completed both pre and posttest administrations of the PSVT:R.

**Table 1.** Sample sizes for both control and intervention sections from each instructor.

<b>Institution and Term</b>	<b>Control</b>	<b>Intervention</b>
WCC Instructor 1	17	38
WCC Instructor 2	11	15
NSC Instructor 1	17	23
TCC Instructor 1	27	47
Total	72	123

Table 2 on the next page presents the population demographics. Students in the study self-reported this information as part of the informed consent process at the end of each course.

**Table 2.** Demographic information of the study population along dimensions of gender, race/ethnicity, and age. All data is self-reported.

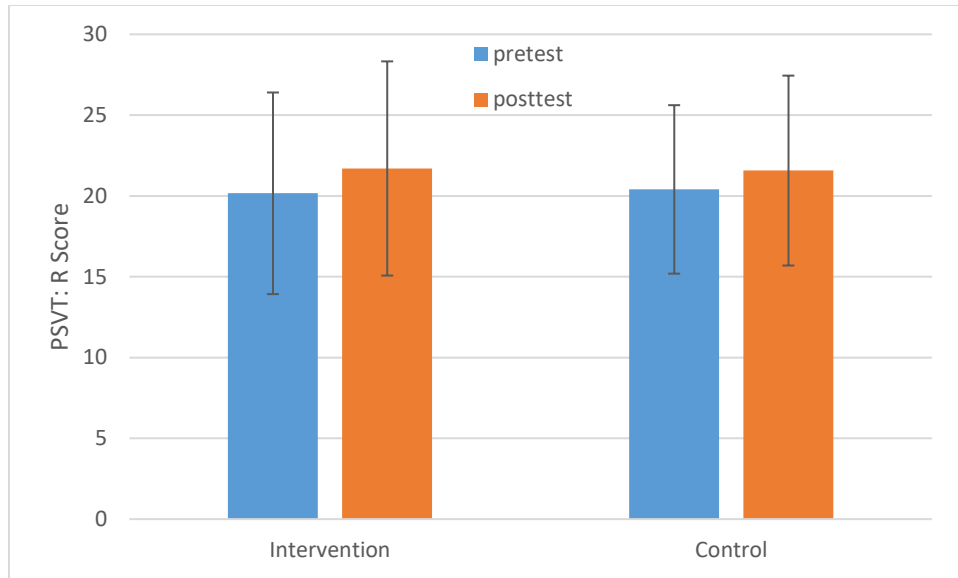
Category	Breakdown	
	Control	Intervention
<b>Gender</b>		
Female	26	43
Male	42	71
Other	4	9
<b>Race/Ethnicity</b>		
Asian or Pacific Islander	13	32
Black or African American	2	3
Hispanic or Latinx	9	4
Native American, Alaska Native or Indigenous	0	0
White or Caucasian	40	69
Multiracial or Biracial	4	8
A race/ethnicity not listed	0	0
Prefer not to answer	2	4
<b>Age</b>		
17 or younger	4	6
18-19	26	34
20-22	22	25
23-29	10	41
30-39	9	16
40 and above	1	1
Prefer not to answer	0	0

## Results and Discussion

### *Spatial Skills (RQ1)*

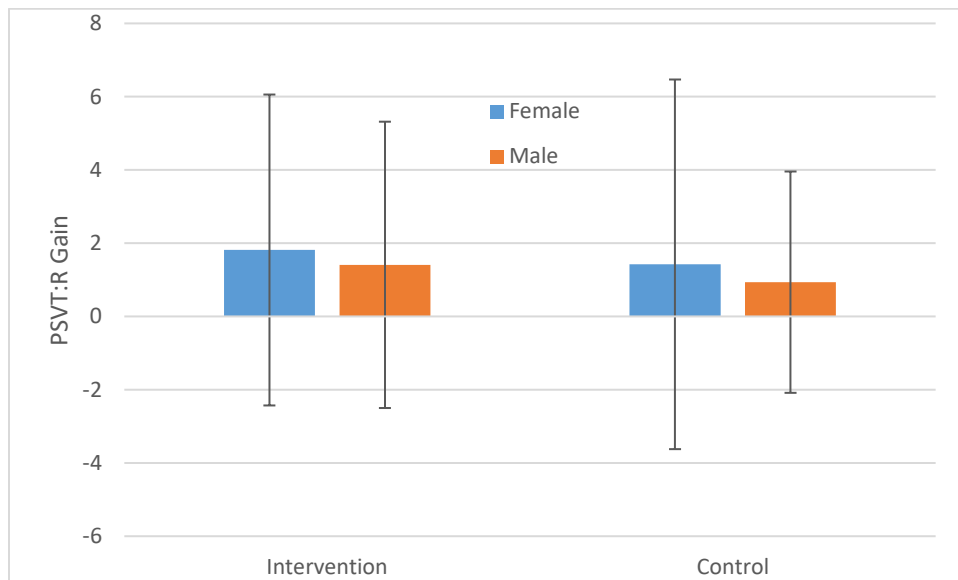
Figure 2 on the next page shows PSVT:R results for the entire study population. The posttest scores are higher for both intervention and control conditions. The average gain of 1.54 in the intervention sections is significant at  $p < .001$  on a paired t-test. The average gain of 1.17 in the control sections is also significant at  $p < .05$ , but the difference between the gains in the intervention and control sections is not significant ( $p = .53$  from un-paired t-test). In both cases, the gain is consistent with what Sorby reported (gain = 1.47) for the comparison population (no spatial skills course) [7]. This gain may reflect typical spatial skills improvement due to engaging with calculus content or may come from more familiarity with the PSVT:R instrument on the posttest administration.

We also note that the initial pretest mean score of 20.25 (67.5%) is lower than the total mean pretest score of 22.51 reported by Sorby [7] and close to the 60% threshold (18 out of 30) that required students in that study to enroll in a concurrent 1-credit spatial skills training course.



**Figure 2.** PSVT:R scores for week 1 pretest and week 10 posttest. Error bars represent  $\pm 1$  SD.

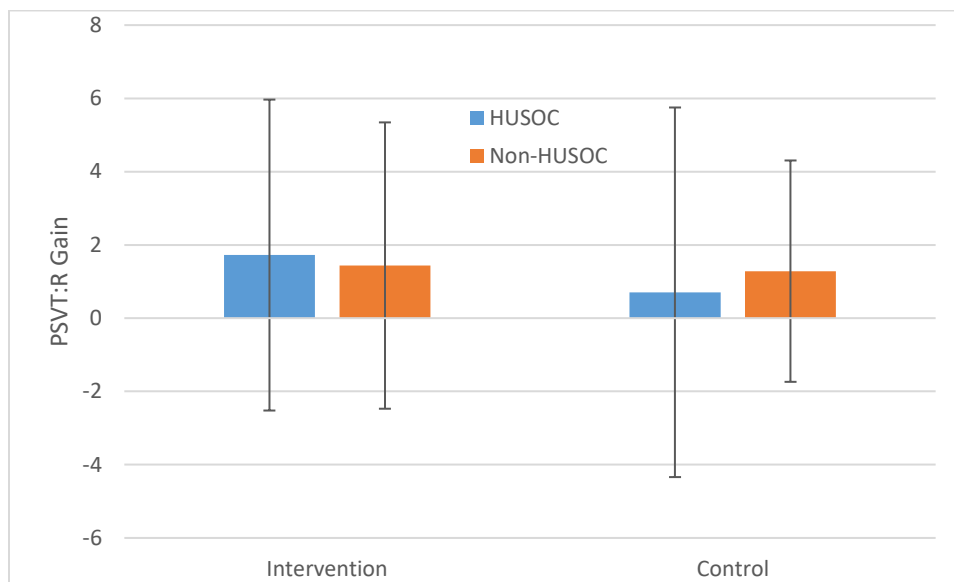
Figure 3 presents gain results with a breakdown along gender. Students who self-reported a gender other than female or male are excluded from this analysis due to their low numbers in the study population ( $N = 13$ ).



**Figure 3.** PSVT:R gain broken down by gender and condition. Error bars represent  $\pm 1$  SD.

While it appears that female-identifying students made slightly larger gains in the intervention sections compared to control sections, this result is not statistically significant ( $p = .53$ ). The difference in gains between genders in both conditions is also not significant. It's possible these differences could become significant with increased sample size.

Figure 4 presents results broken down along race/ethnicity. Due to overall low numbers of these demographics, we have aggregated data for historically-underserved students of color (HUSOC), namely students who identified as black, Hispanic/latinx, indigenous, or multi-racial.



**Figure 4.** PSVT:R gain broken down by race/ethnicity. Error bars represent  $\pm 1$  SD.

Here we see a more pronounced difference with HUSOC student gain of 1.72 in intervention sections versus 0.70 in control sections, but the difference is not statistically significant ( $p = .54$ ). But note there is a low number of these students in the study population ( $N_{\text{control}} = 17$ ,  $N_{\text{intervention}} = 18$ ). It's possible this difference could be significant if the trend persists with increased numbers. Nonetheless, this result along with the potential benefits to female-identifying students provide some evidence that the incorporation of the models may help scaffold spatial skills development for female-identifying and HUSOC students, both populations that tend to score lower on the PSVT:R at college entry [2].

#### *Course Outcomes (RQ2)*

To investigate RQ2, we compared final course grades between the control and intervention sections. We found no significant difference for the aggregate population or along any demographic dimension. For example, the mean numeric final grade in the control sections was 3.28 and the mean grade in the intervention sections was 3.04. The two-tailed unequal variance ttest on these results yields  $p = .287$ . We also tested whether students who scored less than 60% on the PSVT:R pretest performed better when the models were available as learning aids and found no significant difference.

Ultimately, we conclude that final course grades are too coarse a measure to use to evaluate the impact of the models on student learning outcomes for several reasons including the following. First, the intervention only targets a few select content areas, whereas the course grade reflects student performance across the entire term. Second, course grading practices may differ significantly across instructors at different institutions with potentially different departmental norms and policies. And finally, this study occurred over the course of eighteen months of online



learning, with associated dynamics in terms of instructor adaptation to online teaching and student adaptation to remote learning that this study had no mechanism to account for. The following section further discusses some additional study limitations. We look to our future work analyzing the data from the more targeted assessments using multiple-choice plus explain concept questions that are more closely tied to the topics addressed with the models in order to address the question of whether the models are helpful for conceptual learning.

### **Study Limitations**

There are limitations to this study that we should acknowledge. First and foremost, the data was collected in the context of online learning during the COVID-19 pandemic with associated extraordinary stressors on many in the community college student population. Many of the students involved would otherwise be engaged primarily in face-to-face instruction if they had the choice. There was also considerable variability between the four instructors in their overall course design and online pedagogy as well as in their implementation approaches for incorporating the models. We also see significant instructor-level variation in the assessment results that we do not explore in this paper. As mentioned previously, we originally designed and tested the models and activities for group learning in face-to-face classes. Our hasty adaptation to online implementation had multiple shortcomings [11]. The level of student engagement with the models was difficult to monitor and manage in the asynchronous online modality. In fact, one participating instructor reported that several of their students never picked up their model kits at the college bookstore.

### **Conclusions and Future Work**

In summary, students' gains on the PSVT:R in these asynchronous online integral calculus courses are in line with previously-reported results for a standard introductory calculus course. We found no significant effect of the presence of the model-based activities leading to increased gains. We also found no significant impact of incorporating the models on students' final course grades. We would not extend these conclusion to implementation of the curriculum in face-to-face instruction due to the compromises we made to adapt the curriculum from in-person group learning to asynchronous individual work. There was also inconsistent engagement of the online students with the modeling activities that could be more actively managed in face-to-face instruction.

In future work, we plan to further investigate the impact of the models on conceptual learning as evidenced by student performance on a series of multiple-choice plus explain concept questions. We also plan to explore how instructor-level effects may be at play in these results. There is generally wide variation in the PSVT:R results among the participating faculty that may be associated with differences in implementation approaches. We are in the process of conducting post-implementation interviews to gather more information that may help explain these differences.

## Acknowledgement

This material is based upon work supported by the National Science Foundation under grant numbers DUE #1834425, 1834417 and 2022412. Any opinions, findings, and conclusions or recommendations expressed are those of the authors and do not necessarily reflect the views of the NSF.

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