Feel the force! An inquiry-based approach to teaching free-body diagrams for rigid body analysis

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Abstract

Perusal of any common statics textbook will reveal a reference table of standard supports in the section introducing rigid body equilibrium analysis. Most statics students eventually memorize a heuristic approach to drawing a free-body diagram based on applying the information in this table. First, identify the entry in the table that matches the schematic representation of a connection. Then draw the corresponding force and/or couple moment vectors on the isolated body according to their positive sign conventions. Multiple studies have noted how even high performing students tend to rely on this heuristic rather than conceptual reasoning. Many students struggle when faced with a new engineering connection that does not match an entry in the supports table.

In this paper, we describe an inquiry-based approach to introducing support models and free-body diagrams of rigid bodies. In a series of collaborative learning activities, students practice reasoning through the force interactions at example connections such as a bolted flange or a hinge by considering how the support resists translation and rotation in each direction. Each team works with the aid of a physical model to analyze how changes in the applied loads affect the reaction components. A second model of the isolated body provides opportunity to develop a tactile feel for the reaction forces. We emphasize predicting the direction of each reaction component, rather than following a standard sign convention, to provide opportunities for students to practice conceptual application of equilibrium conditions. Students' also draw detailed diagrams of the force interactions at the mating surfaces in the connection, including distributed loadings when appropriate. We use equivalent systems concepts to relate these detailed force diagrams to conventional reaction components.

Targeted assessments explore whether the approach described above might improve learning outcomes and influence how students think about free-body diagrams. Students use an online tool to attempt two multiple-choice concept questions after each activity. The questions represent near and far transfer applications of the concepts emphasized and prompt students for written explanation. Our analysis of the students' explanations indicates that most students engage in the conceptual reasoning we encourage, though reasoning errors are common. Analysis of final exam work and comparison to an earlier term in which we used a more conventional approach indicate a majority of students incorporate conceptual reasoning practice into their approach to free-body diagrams. This does not come at the expense of problem-solving accuracy. Student feedback on the activities is overwhelmingly positive.

Introduction

The process of analyzing a "real-world" system and drawing a free-body diagram is considered one of the most important skills in an engineering statics course, but student mastery rates are often low [1]. Most engineering students gain their first exposure to free-body diagrams in a physics course in the context of particle mechanics. After revisiting concurrent force systems early in a statics course, students encounter rigid-body equilibrium analysis in the context of other new concepts such as moments, support models, and static equivalency. Their approach to drawing free-body diagrams needs to evolve accordingly. They need to develop understanding of reaction forces and internalize the meaning of a couple moment as they progress into more complex problem solving and analysis.

Perusal of any statics textbook will reveal a table of schematic representations of standard supports (e.g. pin, roller, fixed, etc…) with their associated standard reaction components. The takeaway for many students is a heuristic approach to representing reactions on a free-body diagram based on object translation [2]. The following steps summarize this object translation process: first choose a set of reaction components that correspond to the schematic representation in the problem figure; then draw the vectors at the connection point on the isolated body according to their positive sign conventions. Multiple studies have noted how even high performing statics students tend to rely on this practice and noted how sensitive statics learners can be to the surface features of those schematic representations [2], [3], [4]. This reliance on heuristics may have an underlying cause in students' poor understanding of the basic concept of a contact force between interacting stationary objects [5]. Ultimately, many students struggle when faced with a real-world connection or schematic that lacks salient features that they can match to a familiar entry in the supports table. Yet understanding the force interaction at a connection is key statics knowledge for applying mechanics analysis in future courses and engineering practice.

Engineers communicate and apply concepts such as force interactions using a language of multiple representations that include pictorials, diagrams, graphs, symbols, numbers and narrative language [6]. Kozma and Russel [7] used the term representational competence in the context of chemistry education research to describe the ability to use multiple representations of a concept as appropriate for learning, problem solving, and communication. The situation we describe above is an example of students lacking representational competence with the schematics used to communicate engineering connections in mechanics problem figures. In the case of support models, translation from one representation (the schematic) to a new representation (force vectors on a free-body diagram) can be either a heuristic-driven object translation process as described above, or a conceptual reasoning process. Students build conceptual knowledge by thinking through multiple representations and translations. Through this process students resolve misconceptions (or naïve conceptions) and build mental models of the underlying meaning the representations communicate [8]. A representationally competent problem solver coordinates information in both representations with a mental model of how the reaction forces develop to restrict movement based on the geometry of the connection. The introduction of rigid body free-body diagrams in a statics course is an opportunity to emphasize conceptual reasoning about body interactions and force systems with a goal of fostering development of broader fluency with mechanics representations. This opportunity can be lost if we present drawing a free-body diagram as a rote procedure of following conventions.

Toward this goal, we developed an inquiry-based approach with three hands-on activities in which students develop their own process for drawing and analyzing free-body diagrams with the aid of multiple models. Studies in chemistry education demonstrate the promise of hands-on models and manipulatives to serve as scaffolds for novice learners to develop representational competence when working with multiple 2D representations [9], [10]. Our activity design features student teams of 3-4 students completing worksheets with the instructor acting as facilitator and shares many elements with POGIL (Process Oriented Guided Inquiry Learning) [11]. The models serve as learning and communication aids as students discuss and respond to worksheet prompts. Some prompts guide them through a process of exploring how mechanical connections resist motion while other prompts encourage tactile experience by asking students to simulate the effect of a connection with their hand. Students use this experience to inform their process of adding force vectors to their free-body diagrams.

The use of hands-on models and manipulatives in mechanics instruction is not new or novel. There is a consistent strain of this work dating back decades [5], [12], [13], [14], [15], [16], [17], [18], [19], that includes approaches specifically targeting students' understanding of support models [20]. The work by Steif and Dollár in particular serves as inspiration for our approach with its emphasis on using manipulatives to help students develop a tactile feel for forces, couples, and reactions [5], [12]. We seek to build on this prior work by more closely tying the surface features of the manipulatives to typical statics textbook problems and interweaving qualitative analysis and conceptual exploration with tutorials on problem-solving procedures. We hope this connection will translate into greater numbers of students successfully integrating conceptual reasoning into their problem solving practice in homework, rather than compartmentalizing problem solving procedures and concepts as separate areas of knowledge [3]. The guided inquiry process we use links experience with the models to conceptual analysis of force systems in general, all with an eye toward fostering and assessing the representational competence of our students. The following sections provide more detail about these activities, discuss observed patterns in students' reasoning, and compare summative assessment results to a prior term in which we followed a more traditional approach by introducing support models with a lecture.

Learning Activities

This paper focuses on three hands-on learning activities we use to introduce and develop concepts around free-body diagrams and rigid body equilibrium analysis. The activities generally consist of four types of tasks.

- Analyze how the connection of interest restricts kinematic translation and rotation in each direction.
- Use the analysis to add reaction force and/or couple moment components to a free-body diagram.

- Explore how changes in the direction and/or location of the applied loads affect which reaction components develop (i.e. are nonzero).
- Draw detailed diagrams of the force interactions in the connection.

The approach emphasizes how reactions develop in response to the applied loads and in accordance with the ways in which motion is restricted at the connection. We encourage students to predict the direction of reaction components, rather than following a convention of drawing reaction forces in a positive direction relative to the chosen coordinate system. This approach emphasizes the idea of constructing a free-body diagram as a conceptual reasoning process rather than a rote procedure.

The three learning activities we describe here are part of a broader series of hands-on statics activities that we have been developing since 2016 [19]. This curriculum and the associated files for 3D printing the models are available for free download through the Concept Warehouse website [21] and at www.handsonmechanics.com.

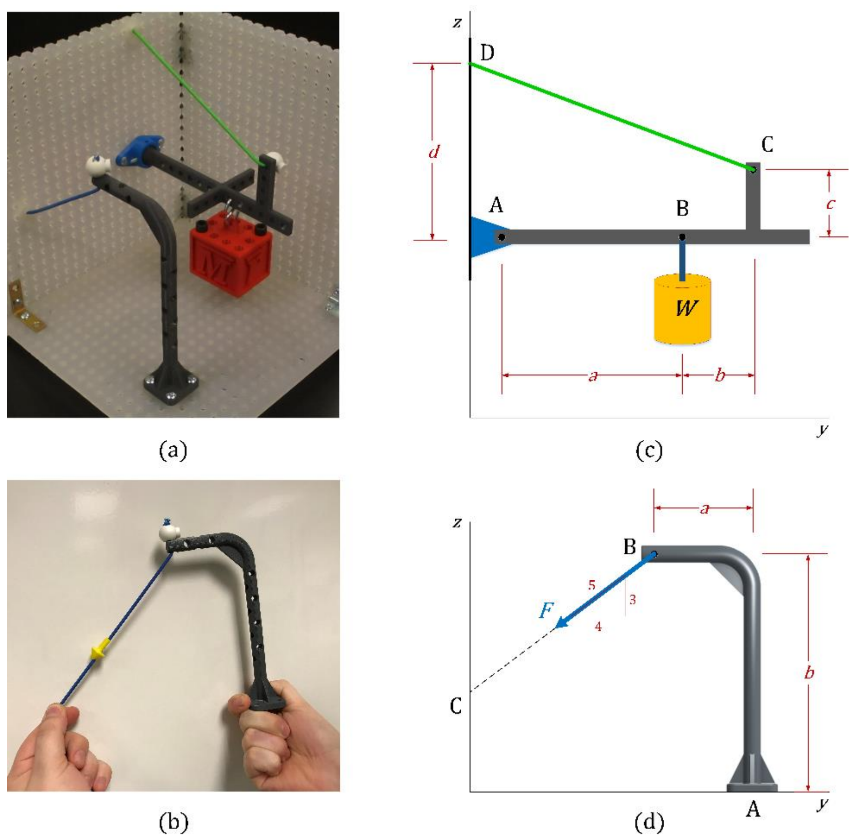


Figure 1. Introduction to free-body diagrams activity. Photos (a) and (b) of models and manipulative representing statics problem figures shown in (c) and (d).

Activity 1: Introduction to Free Body Diagrams

We do not lecture or assign reading to prepare students for this first activity and devote a full 85-minute class session. This activity marks their first exposure to this topic unless they read ahead. The activity introduces free-body diagrams of rigid bodies through consideration of the two statics problems and associated models shown in Figure 1. We also provide students with second copies of the grey beam model and the gray curved post model (see Figure 1b) in addition to the parts installed in the pegboard system shown in Figure 1a. This practice allows students to manipulate the objects they are isolating for analysis as free bodies.

The first task focuses students' attention on the beam in Figure 1c and instructs them to complete the table included here (with correct entries) as Table 1. This exercise stimulates robust discussion within the student groups, generates clarifying questions for the instructor, and prompts many students to manipulate the models to feel how the support restricts motion.

Table 1. Students complete this table in the first exercise of activity 1.

Type of motion at connection point A	Is net force trying to move it? (Y or N) Which direction if Y?	Is motion prevented? (Y or N)
Translation in the x direction.	N	Y
Translation in the y direction.	Y, to the left (negative y)	Y
Translation in the z direction.	Y, down (negative z)	Y
Rotation about A in the x direction.	N	N
Rotation about A in the y direction.	N	Y
Rotation about A in the z direction.	N	Y

Students move on from this reasoning exercise to construct a free-body diagram with the prompt:

> *Add labeled force vectors and/or couple moments to the figure below to complete the free-body diagram (FBD) of beam ABC. Be sure to represent each force as it acts on the beam and add a reaction component (force or couple) for each type of motion the support at A resists (i.e. you answered "Yes" in both columns in the table).*

Next comes a similar analysis of the flange connection of the post base in Figure 1d. After completing a free-body diagram, students consider the couple-moment reaction in the support with more depth. We ask them to think about the concept of static equivalency and "complete the diagram below (included as Figure 2) by sketching the actual force distribution acting on the flange that arises from the contacting bodies of the bolts and the ground."

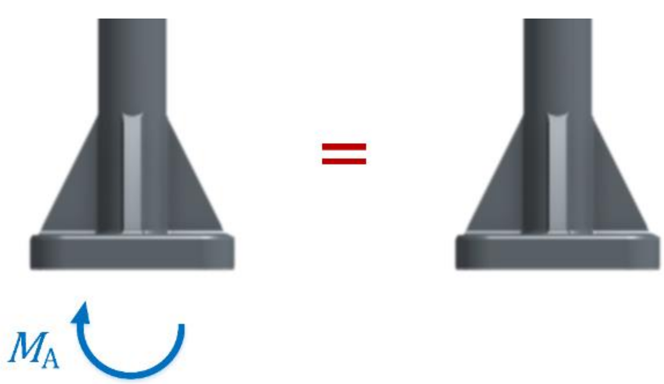


Figure 2. Students draw a force distribution on the right hand diagram to represent how they think the forces of contacting bodies give rise to the clockwise couple moment reaction.

The next section of the activity explores how the reaction components develop or disappear in response to changes in the applied loads, including changes that transform these two problems from 2D to 3D systems. Students continue with the post in figure 1d with the following prompt.

> *What happens to the reaction components if the line of action of the force F changes such that F no longer is parallel to the yz plane?*

They respond to the prompt by completing another table analogous to Table 1 and explain why some of their answers are different. They then draw a new free-body diagram of this 3D system before considering two additional cases communicated with hypothetical numerical values for the applied force: $\vec{\boldsymbol{F}} = 100\hat{\boldsymbol{\imath}}$ lb and $\vec{\boldsymbol{F}} = 100\hat{\boldsymbol{\imath}} + 100\widehat{\boldsymbol{k}}$ lb. The activity concludes by returning to the system in Figure 1c and exploring the pin support in more depth with analogous tasks.

After this session, we assign reading and online videos that include examples with a more conventional heuristic-based approach to free-body diagrams. The next class session features some lecture and group problem solving focused on 2D rigid body equilibrium problems before moving on to the next activity described below. We find that most students continue to try to predict directions of reaction components on their free-body diagrams when completing the worksheet (rather than drawing them all as positive with respect to the coordinate system) and that this practice extends to their approach to other problems. We interpret this trend as evidence of their reasoning efforts and developing representational competence. We have chosen to emphasize and encourage this practice when discussing examples as a class.

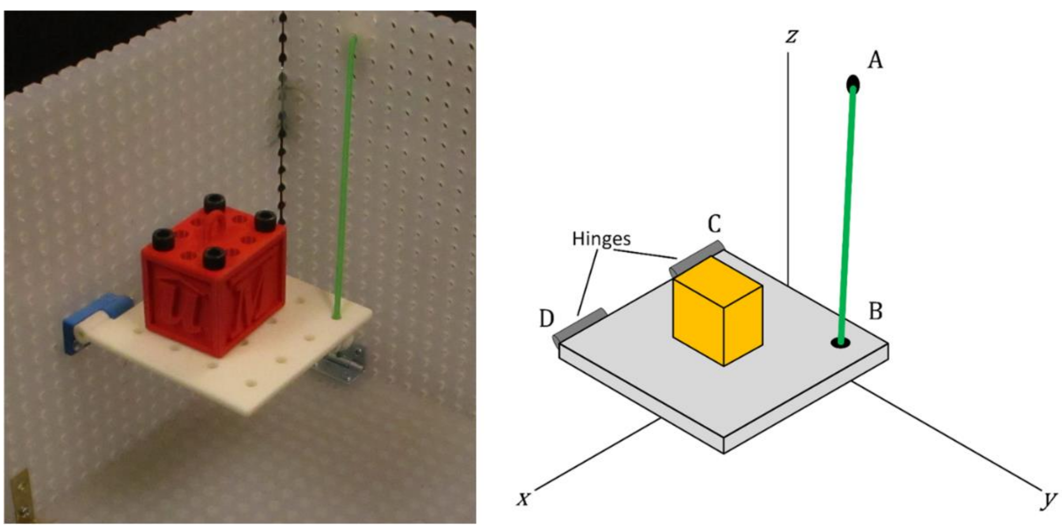


Figure 4. Activity 2: 3D Rigid Body Equilibrium Analysis. Student count holes on the model to read approximate dimensions for quantitative analysis.

Activity 2: 3D Rigid Body Equilibrium

Figure 4 shows the model and associated statics problem figure that forms the basis for the second activity on 3D rigid body equilibrium analysis. This activity requires about an hour, most of the third class meeting. It features a somewhat different table (shown below in Table 2) to guide students through the process of determining the reaction components that develop in the two hinges now that the concept of a reaction forces and couples are more formally defined.

Students draw a free body diagram of the platform after reasoning through the table. Again, we provide students with an extra copy of the isolated body (i.e. the platform) to manipulate and develop a tactile feel for the forces that develop in the hinges. In contrast to the previous activity that focused exclusively on conceptual exploration, the next section of this worksheet guides students step-by-step through a quantitative analysis to formulate the six scalar equilibrium equations for this system.

Table 2. Example solution of the table students complete as they reason through the reaction forces in the hinges in Figure 4.

Reaction Component	Hinge C		Hinge D	
	Reaction Possible	**Developed?**	**Reaction Possible**	**Developed?**
Force x-component	Y (+ x only)	Y	Y (-x only)	N
Force y-component	Y	Y	Y	Y
Force z-component	Y	Y	Y	Y
Couple x-component	N	N	N	N
Couple y-component	Y	N	Y	N
Couple z-component	Y	N	Y	N

The last section of the worksheet prompts students to analyze how the reactions must change to maintain equilibrium when they remove either hinge. Students explore how a reaction couple must develop when only a single hinge is present and reflect back on how the two hinges work together to generate an equivalent couple in the previous configuration. They also consider and discuss how changing the direction of the cable tension would affect the system.

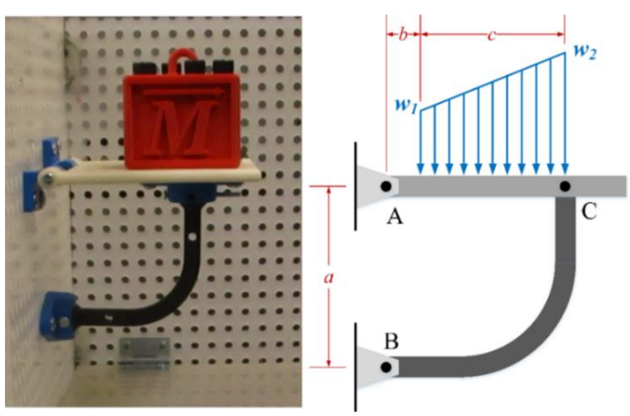


Figure 5. Activity 3: Two and Three-Force Members.

Activity 3: Two and Three Force Members

Figure 5 shows the last activity in this module on rigid body equilibrium analysis. This activity usually takes about an hour. Students construct a free-body diagram of member BC. They have a second copy of this model. The model has spherical ends at B and C. We instruct students to hold the piece with one finger on either end so they can feel how the forces at B and C must be collinear to maintain stable equilibrium.

They draw a free-body diagram and perform a quantitative analysis to solve for the forces on member AC. Then we ask them to perform some conceptual analysis of the problem and consider how they can use the fact that AC is a three-force member to reason through the following prompt:

> *Suppose the dimension a increases but all other dimensions are held constant. Complete the table below to explain how the forces acting on the platform would change. Use **I** for increase, **D** for decrease, and **U** for unchanged.*

Table 3. Completed table in Activity 3 example solution. The direction column is an angle measured counterclockwise from horizontal.

Force	Magnitude	Direction	Notes
Reaction at A	D	D	Horizontal component decreases as direction rotates CW to maintain intersection with line of action of force at C.
Reaction at C	D	I	The direction rotates CCW to stay concurrent with line BC. The direction is more vertical so less force needed to counteract the moment of the applied load.

Some of the qualitative analysis tasks we describe here are quite difficult for many students. This is intentional. Through piloting these activities in focus groups and in prior versions during class, we have learned that many students will not engage with the models (e.g. touching and manipulating) if they can answer the questions without doing so. Our goal here is not so much that they accurately perform each task, as it is to provide context and motivation for the students to think conceptually about the force systems in these problems, and subsequently transfer that conceptual reasoning practice to their work on homework and exams. We discuss the extent to which we have observed that transfer in the assessment section below.

Implementation Context

This paper includes data from two successive quarters of implementation in the engineering statics course at a community college. The class meets for three 85-minute sessions per week over a ten-week academic quarter and has a maximum capacity of 24 students. Table 4 summarizes student enrollment, demographics, and academic preparation of the study population.

In all cases, the gender, race/ethnicity, and first generation status data are self-reported. None of the differences in the demographics or academic preparation are statistically significant, though that is largely an artifact of the small sample sizes. We should note that the mean STEM GPA for the winter 2020 cohort is substantially lower, if not significantly so. Nonetheless, we combine the fall 2019 and winter 2020 cohorts into one sample in the analysis that follows with a couple noted exceptions. The table includes information from winter 2018 as well which we use for comparison in some of the analysis that follows. That section pre-dates the specific inquiry-based approach described here but did include some hands-on modeling exercises on the topic of rigid body equilibrium [19].

Table 4. Demographics data for the statics sections referenced in the analysis below.

Section	Fall 2019	Winter 2020	Winter 2018 (Control)
Number students enrolled	14	17	25
Historically Underserved Students of Color	11%	20%	31%
Female-identifying	29%	12%	13%
First generation college students	21%	35%	36%
Mean STEM GPA	3.50	2.98	3.20
Mean successful STEM credits completed	42.1	46.1	34.5

The three activities described above account for the majority of class time during a module on rigid body equilibrium that consists of four class meetings covering the topics of 2D and 3D rigid body equilibrium, static determinacy, and two- and three-force members. These sessions also feature small amounts of lecture, some concept-based peer instruction, and group problem solving on whiteboards. Students are already familiar with some of the model components, figure conventions, and the POGIL-inspired class session design because they explored vector concepts and moments through analogous activities earlier in the term. Not all student groups finish the activity worksheets during the allotted class time. The models are available for students outside of class during open lab hours and instructor office hours. Students submit completed worksheets with a deadline generally set at midnight of the day following the activity. They also must complete two multiple-choice concept questions in order to earn full credit. We administer these questions online using the Concept Warehouse [21] with prompts for students to explain their answer choices. The instructor grades both the worksheets and the associated question responses for completion only and reviews them as part of their formative assessment process to identify emphasis areas for the discussion in future class meetings. We provide example worked solutions for each worksheet through the learning management system that students can use as study aids.

Assessment Results and Discussion

In developing our assessment strategy, we are primarily investigating the following research question:

> *Do students successfully transfer the conceptual reasoning practices emphasized in the activities to other contexts (e.g. course exams)?*

Recall that students still encounter the more heuristic-oriented approach through the various resources they use outside of class time, including multiple examples they consult while working through assigned problem sets. Ultimately, they encounter multiple approaches to free-body diagrams and we are curious to see which they pursue. We are also interested in whether this approach might help or hinder student progress on course learning outcomes.

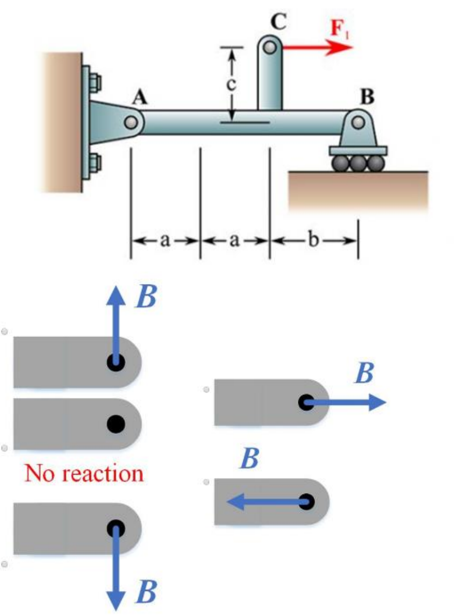
Analysis of Worksheet Follow-up Questions

As mentioned previously, we assign two concept questions with each activity. We analyzed the written explanations to look for themes in students' reasoning using a process similar to the open coding approach of Koretsky et al [22], though less rigorous with only one coder. Figure 6 shows the follow-up questions for the first activity on free-body diagrams and supports. These questions as well as the follow-up questions for the other two activities are all freely available "Concept Tests" in the question library on Concept Warehouse and identified by item number in the activity descriptions where the worksheets and model files are available for download in the "Instructional Tools" section of the website.

Activity 1: Follow-up question 1

The beam is subjected to the load F_1 at C and supported by the smooth pin connection at A and a roller at B that allows horizontal translation.

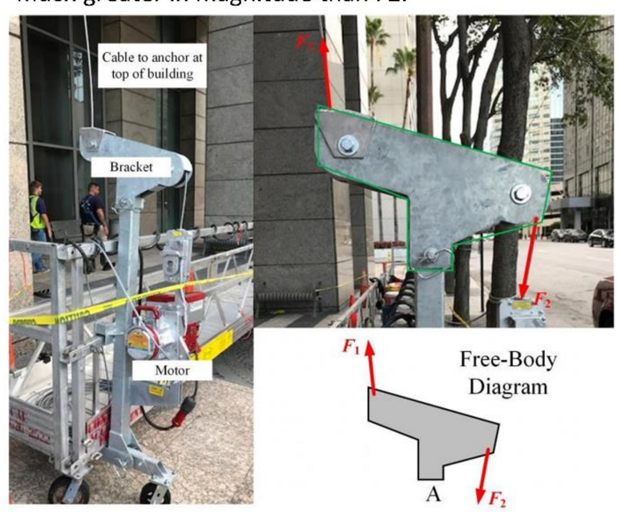
Which diagram most accurately depicts the force the support at B exerts on the beam?



Activity 1: Follow-up question 2

The figure shows a platform for exterior window washing on high-rise buildings. The cable runs through a pulley system in the bracket to a motor-driven winch system that allows the workers to raise and lower the platform.

You are analyzing the loads on the bracket and need to complete the free-body diagram in the figure. What support reactions develop at point A in response to the loadings F1 and F2? Assume F1 is much greater in magnitude than F2.



- Force components down and to the right. No couple moment.
- Force components down and to the right. Counterclockwise couple moment.
- Force components up and to the right. Counterclockwise couple moment.
- Force components down and to the right. Clockwise couple moment.
- Force components up and to the left. Clockwise couple moment.

Figure 6. Follow-up questions for the free-body diagram activity. Student explanation prompts removed.

We combined fall 2019 and winter 2020 student responses for the following analyses of student explanations. For question 1 (A1Q1), 18 of 27 (67%) responding students chose the correct answer. For question 2 (A1Q2) 13 of 25 (52%) responding students chose the correct answer. For each question, we coded student explanations for evidence of conceptual reasoning regarding equilibrium conditions and/or rote application of heuristics (or a perceived general rule). Table 5 summarizes these results. Most students (18 of 27 on A1Q1 and 23 of 25 on A1Q2) attempted to apply conceptual reasoning in explaining their answer choices, even if they did not arrive at the correct response. This result is unsurprising because they attempted these questions shortly after completing activities in which the instructor emphasized this reasoning approach.

Table 5. Analysis of student explanations for the follow-up to the first activity. Includes fall 2019 and winter 2020 data.

		Example Student Explanations
A1Q1	18 students attempted to apply reasoning (12 correct answers)	*The force at B will need to counter the moment that the force could cause.* (correct answer choice) *Because the F1 on point C will cause the beam to rotate to the clockwise direction, so it's pushing the beam down to the point B. So, the force of support point B exert to the beam will be the reaction force: going upwards.* (correct answer choice) *Because the A is one that that reaction because of the pin but there is no reaction B. if for example, we take the pin the system will move to the right.* (incorrect answer choice)
	7 students attempted to apply a heuristic (6 correct answers)	*The resulting reaction force for a roller support is always a single force that is perpendicular to, and away from, the surface.* (correct answer choice) *It's the normal force the floor exerts to the roller.* (correct answer choice) *Forces do not change direction when moving them, only the moment reaction changes.* (incorrect answer choice)
A1Q2	23 students attempted to apply reasoning (12 correct answers)	*force resultant is up and to the left and causes it to rotate cw.* (correct answer choice) *Because F1 is a much greater force it will cause a moment in the clockwise direction, so there must be a reaction couple moment in the CCW direction. The resultant force will be up and to the left since F1 is much larger, so there must be a reaction force down and to the left at point A.* (correct answer choice) *Down and right allow for some of the forces to equal 0 since we are provided f1 is much greater than f2. CCW couple moment since the structure wants rotate CW.* (correct answer choice)
	2 students attempted to apply a heuristic (0 correct answers)	*It's stated that F1's magnitude is much greater than F2 so we can disregard the effect of F2 and focus on F1. A reaction at A would be equal and opposite of F1. It's my understanding that a single force cannot have a couple moment so it wouldn't have a couple but I'm still unsure on certain applications of couples.* (incorrect answer choice) *The point at A has a pin through it which does not the resist any moments due to forces F1 and F2.* (incorrect answer choice)

Table 6. Analysis of student explanations for the follow-up to the second activity. Includes fall 2019 and winter 2020 data.

`		Example Student Explanations
A2Q1	11 students attempted to apply reasoning (7 correct answers)	*Force pushing up to counter sign weight, force pushing against wind, moment due to sign weight, moment due to wind on pole, moment due to wind on sign.* (correct answer choice) *Potential for translation in the x and z directions. Potential for rotation in the x, y, and z directions.* (correct answer choice) *There'll be 4 non zero components on the base because there'll only be 2 unknowns from the forces which is the weight of the billboard, and the wind pressure. The other two comes from the moments because we have 2 axis to consider the moment about.* (incorrect answer choice)
	3 students attempted to apply a heuristic (0 correct answers)	*the base is fixed support so it has 3 component forces.* (incorrect answer choice) *As the system is in statics, there is reaction forces and moments that counter act the wind pressure. Those forces and moments are all in x,y,z axis.* (incorrect answer choice)
A2Q2	10 students attempted to apply equilibrium in their reasoning (4 correct answers)	*As the box is moved it will cause an increase of moment reaction since the platform will try to rotate.* (correct answer choice) *The moment reaction has no X component because it is a hinge. The Z component is caused by the tension, which does not change. The Y component is caused by the weight of the box, and as the box moves, the moment there increases. If the Y component of the couple increases and the others stay the same, both direction and magnitude are changing.* (correct answer choice) *The box only causes a y-moment (x-moments are not resisted). Moving the box in the negative x direction causes a greater distance, which causes a greater moment.* (incorrect answer choice)
	16 students only included moments in their reasoning (15 incorrect answers)	*The box is increasing in distance away from the hinge, increasing the magnitude of the moment.* (incorrect answer choice) *Distance to the box becomes very far, thats why the magnitude increase but the direction stays the same.* (incorrect answer choice) *Moving the box in the negative x direction just makes the moment arm bigger.* (incorrect answer choice)

The follow-up questions for activity 2 focused on support reactions in 3D rigid body equilibrium analysis. Question 1 (A2Q1) asks students to count the number of nonzero reaction components develop in the fixed support base of a road sign subject to weight and wind loadings. Question 2 (A2Q2) refers back to the problem figure in the activity (see Figure 5 above) but with only one hinge and asks how the resultant couple moment reaction in the hinge would change if the box moves in the negative x-direction. The answer choices are as follows:

- o The magnitude increases but the direction stays the same.
- o The magnitude increases and the direction changes.
- o The magnitude decreases but the direction stays the same.

- The magnitude decreases and the direction changes.
- The magnitude and direction both stay the same.

Both of these questions proved exceptionally difficult. Only 9 of 27 (33%) responding students correctly answered A2Q1. Only 6 of 27 (22%) students answered A2Q2 correctly, but most correctly identified that one component of the couple moment reaction would increase and failed to make the connection that this would change the direction of the resultant couple. Despite their difficulty, these questions still proved useful for gaining insight into students' reasoning. Table 6 summarizes our analysis of the students' explanations. These two questions provided context for robust class discussions about the relevant concepts during the following class meetings.

The follow-up questions for the third activity focus on two and three-force members. A correct response to the first question (A3Q1) requires identification of a two-force member and observation of the fact that the force it exerts at a connection will not change direction when new forces are added to the structure. 16 of 24 (67%) students answered A3Q1 correctly. The second question revisits the system in A1Q1 but focuses attention on the resultant reaction force that develops in the pin at A:

> *How will the resultant reaction force at A change if the dimension c increases? Assume the magnitude of F_1 does not change.*

Students choose from the following answers:

- The magnitude will increase but the direction will stay the same.
- The magnitude will be unchanged, but the direction will rotate clockwise.
- The magnitude will be unchanged, but the direction will rotate counterclockwise
- The magnitude will increase and the direction will rotate clockwise
- The magnitude will increase and the direction will rotate counterclockwise.

Only 4 of 23 (17%) responding students chose the correct answer, but most students correctly identified that the magnitude would increase. Observing that the force at A would rotate to maintain the concurrency of the force system proved difficult. Unfortunately, a technical problem prevented us from collecting and analyzing students' explanations for this set of follow-up questions for fall, so Table 7 on the next page only includes results from winter 2020 students.

In summary, the assessment data we collected with these concept questions indicates that most students attempted to apply the reasoning approaches we introduce in the activities and model in the accompanying short lectures. There did not seem to be heavy reliance on heuristics or memorized rules in answering these questions. Their success at applying that reasoning accurately was mixed. This result is unsurprising because students answered these questions in the context of the activities that emphasized reasoning over heuristics. To address the research question, we turn our attention to analysis of student work on the course exams to see if this practice transferred beyond the context of the activities.

Table 7. Analysis of student explanations for the follow-up to the third activity. Winter 2020 data only.

`		**Example Student Explanations**
A3Q1	5 students attempted to apply equilibrium reasoning (3 correct answers)	*Putting a force between B and D pointing right and downwards would add to the moments at B. If the moment at B increases the supports will have more force applied top them because they are not allowing anything to rotate.* (correct answer choice) *Before the moment was applied, the force was up and to the left, but once the moment is exerted, the push from the moment causes the reaction force to reverse and increase.* (incorrect answer choice)
	2 students attempted to apply a heuristic (1 correct answer)	*The direction cannot change because it is a two-force member. I have no idea about the magnitude, I just figure it has to increase somewhere.* (correct answer choice)
A3Q2	4 students attempted to apply equilibrium in their reasoning (0 correct answers)	*Since the sum of the forces in the x direction is the same only the couple forces at A and B change making the vector at A have a larger negative y component.* (incorrect answer choice) *As c increases, the moment created by F1 increases too. In order to create a couple to cancel it, the reaction at A must increase in the negative Y direction.* (incorrect answer choice)
	2 students attempted to apply a heuristic (1 correct answers)	*Because the force direction hasn't changed.* (incorrect answer choice) *Moving the box in the negative x direction just makes the moment arm bigger.* (incorrect answer choice)

Midterm Exam Problem

We only conducted this analysis for the fall 2019 section because the COVID-19 transition to online learning occurred the day before the winter 2020 midterm and resulted in a change in the exam format. Our second assessment is a two-part conceptual free-body diagram problem that we included on the first exam in fall 2019. Part (a) presents students with a cantilevered object with one end embedded in a concrete wall and asks students to draw a "free-body diagram that indicates the correct direction of all reaction components." Part (b) has the following prompt analogous to some of the reasoning we asked students to engage in on the worksheet and follow-up questions.

> *Explain the effect that increasing the angle θ would have on the magnitude of each reaction component at A. Does each component increase, decrease, or stay the same? Why?*

The angle θ specifies the direction of one of the applied loads in the problem figure. We did not include additional details of this exam problem here due to concerns about exam content security. For fall 2019, the mean score on the exam was 64/100 (SD = 22, N = 14). We were not able to include two of the exams in the detailed analysis of student work on this problem due to a duplicating error. The mean score for part (a) of the conceptual free-body diagram problem was 3.3/5 (SD = 1.4, N = 12). The most common errors were incorrect direction of the couple moment reaction at the fixed support, or failure to include the couple moment reaction at all. The mean score for part (b) was 3.2/5 (SD = 1.9, N = 12). Two students who struggled with part (a) did not attempt part (b) and scored zero. The couple moment reaction again appears to be the main difficulty in this problem, not surprising because of difficulties recognizing its presence in the first place in part (a). Only two students were able to complete both parts of this problem with 100% accuracy in a test situation. The overall mean score on this problem of 6.5/10 is in line with the overall exam average, so we do not see it as an overly difficult problem. We conclude that student success in applying conceptual reasoning to support reactions and free-body diagrams is on par with their gains in other aspects of the course outcomes that the exam assessed.

Final Exam Free-Body Diagrams

We only conducted this analysis for the fall 2019 section because the COVID-19 transition to online learning changed the exam format for winter 2020. The last assessment is an analysis of students' free-body diagrams and problem-solving work on the final exam. We reviewed student free-body diagram work for three problems and did the following. First, we counted the instances when students accurately predicted the directions of a reaction component that did not align with a positive coordinate system axis. We call this approach a concept-driven free-body diagram. Next, we counted substantive mistakes on the diagrams that propagate to errors in the associated problem solving work. Here we are trying to determine if student effort at the concept-driven approach is leading to increased errors in problem solving. The following paragraphs provide some more detail on each problem and what we consider evidence of a "concept-driven" approach to developing the free-body diagram.

The first problem was a shear and bending moment diagram for a simply supported beam. We looked specifically at whether students included an x-component of reaction at the pin in their initial free-body diagram. A heuristic-based approach would include the x-component as part of the object translation process of coordinating the schematic representation of the pin with the standard support reaction components for a pin. A more concept-driven approach would identify that the loads are all vertical, apply reasoning focused on force equilibrium to determine that this x-component would be zero, and not include it on the diagram at all.

The second problem was a rigid body equilibrium problem that also requires computation of a centroid. The object hangs from a pin and a spring with only gravitational applied loads. The horizontal reaction component at the pin is readily apparent by inspection to be in the negative x-direction. A heuristic-based approach would adhere to convention and still draw this vector in the positive x-direction. A concept-driven approach would identify the direction in the process of developing the free-body diagram and draw it negative.

The third problem was a simple frame with a beam supported by a pin and a two-force member. Again, we looked at how students drew the horizontal component of the pin reaction. A concept-driven approach to developing the free-body diagram for this problem would identify the two-force member as subject to compression and apply a chain of reasoning based on force equilibrium to draw the pin reaction (the only opposing force in the horizontal components) in the negative x-direction. A less obvious chain of reasoning based on moment equilibrium would indicate the y-component of the pin reaction is also negative.

We conducted this same review on saved final exams from a prior course offering (winter 2018) that pre-dates our use of the approach described in this paper. The winter 2018 section had identical topic coverage, assigned near-identical problem sets, and shared many pedagogical elements including class activities with hands-on models. The only substantive difference between the offerings is the specific inquiry-based approach to introducing free-body diagrams. We introduced support models with a traditional lecture in winter 2018. Table 7 presents the results of the analysis for concept-driven free-body diagrams. Regarding free-body diagram errors, for fall 2019, we counted four free-body diagram mistakes across 14 students attempting three problems, yielding an error rate of 9.5%. For winter 2018, we counted 10 free-body diagram mistakes across 23 students attempting three problems each, yielding an error rate of 14.5%. The error rate reduced by a third, but this result is not statistically significant due to the low number of students in the study. There was also no significant difference in overall final exam scores or scores on the Concept Assessment Test in Statics (CATS) [23], [24].

Table 7. Analysis of free-body diagrams on final exams.

	Work Indicates Concept-Driven Approach	
FBD Analysis Criterion	W18 (N = 23)	F19 (N = 14)
P1. Pin reaction x-component omitted	43%	79%
P2. Pin reaction in negative $\boldsymbol{x}$-direction	55%	75%
P3. Pin reaction in negative $\boldsymbol{x}$-direction	33%	73%
P3. Pin reaction in negative $\boldsymbol{y}$-direction	17%	27%

These results indicate that most of the fall 2019 students successfully incorporated some conceptual application of equilibrium concepts into their approach for drawing free-body diagrams. We further conclude that these gains did not come at the expense of problem-solving accuracy.

Student Feedback

Students provided feedback on the series of three activities on the day following submission of the third worksheet. Table 8 presents the feedback prompts and associated results. The survey uses a six-point Likert scale of 1 = Completely Disagree, 2 = Somewhat Disagree, 3 = Slightly Disagree, 4 = Slightly Agree, 5 = Somewhat Agree, and 6 = Completely Agree. We administer the survey outside as a Google form that students access through a link in the course learning management system outside of class time. Students earn a negligible number of participation points as incentive for completion. Overall, the feedback on the activities was favorable on all prompts. Students especially agreed with the ideas that the models are helpful in communication with classmates and in interpreting the figures in the worksheets. The only prompts that elicited any disagreement were prompt 6 regarding the follow-up questions (1 student slightly disagreed) and prompt 9 regarding how well the experience with the models transfers to interpretation of figures and diagrams in other contexts (2 students slightly disagreed).

We should note an important caveat in interpreting students' positive response to these activities. Faculty at our college, particularly in the STEM disciplines, generally accept findings that active learning is effective [25]. Students are likely to have encountered a variety of active learning pedagogies in multiple classes before they arrive in our statics course. We do not generally experience the resistance to active learning commonly reported in other educational contexts [26]. In fact, responses on end-of-course evaluations generally indicate our students expect and embrace active learning as standard instructional practice.

Table 8. Aggregate feedback survey results for fall 2019 and winter 2020.

Survey Prompt	F19 and W20 (N = 29)	Number of students responding 5 or 6
1. I understood what I was being asked to do.	5.31	25
2. The learning goals for each activity were clear.	5.52	28
3. The models helped me communicate with my classmates.	5.69	26
4. The activities helped me clarify the material we are learning.	5.34	27
5. The models helped me feel the forces and moments in the problems.	5.52	26
6. The follow-up questions on Concept Warehouse helped me test my understanding.	5.17	22
7. The activities helped me connect different representations of the concepts (i.e. figures, diagrams, graphs, notation, equations, written descriptions, etc.)	5.38	27
8. The models helped me visualize and interpret the figures and diagrams on the worksheets.	5.69	29
9. Working with the models helps me visualize and interpret other figures and diagrams in the reading and problem sets.	5.45	26

As mentioned previously, the activities and models we describe in this paper are part of a broader curriculum that spans the first eight weeks of the quarter and covers several other statics topics. Our end of quarter anonymous feedback survey includes the following two prompts:

- Were any of the activities the key contributor to your learning of a specific statics concept?
- If you answered yes or maybe to the previous question, please list the statics concept(s) you are thinking about and briefly explain how the modeling activities helped?

Over four weeks passed between these rigid body equilibrium activities and the administration of this survey, yet six of fourteen respondents identified support reactions as a key concept that the experiences with the models helped them understand. The list below provides a few example quotes from students' responses that specifically mentioned how tactile manipulation of the models helped them understand how support reactions develop.

- "Support reactions in R3. Moving the box around and holding the supports in my hand and twisting or pushing with the other hand"
- "Force and moment reactions: When we applied a force on the hinge, we can feel how it will react to counteract the rotation."
- "3D supports and reactions. Being able to see and feel forces and moments helped develop an understanding of how supports behave."

These survey responses indicate that some students continued to think about their experiences with the models as the course progressed and appreciated the opportunity to "feel the forces" that were the focus of numerous calculations in problem sets and exams throughout the term.

Conclusion

This paper presents an inquiry-based approach to teaching free-body diagrams and rigid body equilibrium analysis in an engineering statics course. A series of three hands-on learning activities and follow-up questions emphasize conceptual reasoning throughout the process of drawing a free-body diagram. Analysis of student explanations on the follow-up questions and work on exams indicates the majority of students are willing to engage with this approach as opposed to relying solely on heuristics as others have observed in studies of student problem solving in statics. Most students correctly applied the approach by the end of the term as evidenced by their free-body diagrams on the final exam. We did not observe any significant differences in student performance on summative assessment measures such as the final exam or CATS, indicating that increased emphasis on conceptual reasoning did not come at the expense of procedural accuracy in problem solving. One potential avenue for future research is to investigate whether the approach we have taken leads to a deeper and more durable understanding of static equilibrium and force interactions that improves learning in follow-on courses such as mechanics of materials.

We acknowledge that our students are probably more receptive than average to active learning. Our course also has a smaller student-to-instructor ratio and more contact time than most statics courses. These circumstances make it easier to implement the curriculum we describe here. While it may be difficult or impractical to replicate this approach in larger classes with shorter meeting times, we believe that the ideas presented here can be adapted to other educational contexts with positive results for student learning.

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